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Continuity of Symmetric Stable Processes

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The path continuity of a symmetric p-stable process is examined in terms of any stochastic integral representation for the process. When $0 < p < 1$, we give necessary and sufficient conditions for path continuity in terms of any (every) representation. When $1 \leqslant p < 2$, we extend the known sufficiency condition in terms of metric entropy and offer a conjecture for the stable version of the Dudley–Fernique theorem. Finally, necessary and sufficient conditions for path continuity are given in terms of continuity at a point for $0 < p < 2$. © 1989 Academic Press, Inc.

1. INTRODUCTION

A real-valued stochastic process $X = \{X(t), t \in T\}$ on an arbitrary index set T is called stable if every finite linear combination $\sum a_j X(t_j)$ has a stable distribution, e.g., Feller [3, VI.1]. During the past two decades there has been a considerable amount of interest in stable processes, in part because they are a natural generalization of Gaussian processes. Some of the stable results are identical to the corresponding Gaussian ones, some are quite different. In this paper we are concerned with the continuity problem for stable processes: when does X have a version with continuous paths.

In this paper, we consider real, symmetric, separable in probability p-stable processes, $0 < p \leqslant 2$, on a compact metric or pseudo-metric space (T, τ). Such processes always have a stochastic integral representation [3], i.e., have a version X given by

$$X(t) = \int_U f(t, u)\, W_m(du), \tag{1.1}$$

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where $(U, \mathscr{U}, m)$ is some sigma-finite measure space, $f\colon T\times U \to \mathbb{R}$ is a function with the property that for each $t\in T, f(t,\cdot)\in L^p(U, \mathscr{U}, m)$, and W_m is the p-stable noise generated by m. Conversely, given any $(U, \mathscr{U}, m)$ and any kernel $f(t, u)$ with $\{f(t,\cdot),\ t\in T\}\subset L^p(U, \mathscr{U}, m)$, (1.1) defines a p-stable process X. It is a basic fact [3, p. 386] that the joint characteristic function of X is given by

$$E\exp\left(i\sum_{j=1}^{n} a_j X(t_j)\right)=\exp\left(-\left\|\sum_{j=1}^{n} a_j f(t_j,\cdot)\right\|^p_{L^p(U,\mathscr{U},m)}\right). \tag{1.2}$$

Therefore if $g(t, u')$ is any other representation for X with $\{g(t,\cdot),\ t\in T\}$ a subset of some $L^p(U', \mathscr{U}', m')$,

$$\left\|\sum_{j=1}^{n} a_j f(t_j,\cdot)\right\|_{L^p(U,\mathscr{U},m)}=\left\|\sum_{j=1}^{n} a_j g(t_j,\cdot)\right\|_{L^p(U',\mathscr{U}',m')}. \tag{1.3}$$

Since such norms (quasi-norms when $0<p<1$) are independent of the representation (1.1), we may use the expression $\|\sum a_j X(t_j)\|_p$ for the terms in (1.3). Note that in the Gaussian case $(p=2)$, $\|\sum a_j X(t_j)\|_2^2 = \frac{1}{2}\operatorname{Var}(\sum a_j X(t_j))$.

Let X and Y be p-stable processes, $0<p\leqslant 2$, and suppose $\|\sum_{j=1}^n a_j X(t_j)\|_p \approx_{c(n)} \|\sum_{j=1}^n a_j Y(t_j)\|_p$, i.e., the ratio of both sides is bounded above and below by a finite, positive number $c(n)$ that depends only on n. At least for a large class of processes, this condition forces the paths of X and Y to have the same degree of irregularity, e.g., [8, Section 4; 9, Corollary 4.3]. In the Gaussian case, it also forces X and Y to be mutually continuous or discontinuous, but when $p<2$, this is not the case [8, (3.8)]. So the continuity problem is more subtle when $p<2$ than in the Gaussian case.

Rosinski [10] has shown that the paths of X are related to the paths $f(\cdot, u)$, $u\in U$, of the kernel in (1.1). An apparent difficulty with this is the non-uniqueness of representations. Under the separability assumption, it is always possible to take the unit interval with Lebesgue measure as our base space, but we have no idea what the kernel function is or how it relates to other representations. Or, if we start with a particular kernel and define X through (1.1), what can other representations look like? Theorem 5.1 of [10] gives necessary conditions for path continuity in terms of any kernel $f(t, u)$, showing there is a rigidity in the possible representations. These ideas are combined with earlier sufficient conditions for path continuity from Marcus and Woyczynski [7] and from Marcus and Pisier [5, 6], which we rephrase in terms of a kernel $f(t, u)$.

In Section 2 we show how the above ideas solve the continuity problem when $0<p<1$. Necessary and sufficient conditions are given for continuity in terms of any (every) representation (1.1) as part of a more general result showing there is a trichotomy on what kind of trajectories stable processes possess when $0 < p < 1$. Section 3 considers the cases when $1 \leqslant p < 2$. We extend the sufficiency results for continuity in terms of metric entropy and conditions on any representation and conjecture what the correct stable analog of the Dudley–Fernique theorem for Gaussian processes is. We end with necessary and sufficient conditions for path continuity in terms of continuity at each point in Section 4.

2. Continuity and Boundedness when $0 < p < 1$

Let X be a p-stable process given by (1.1) with kernel $f(t, u)$. A kernel $f_0(t, u)$ is a *modification* of $f(t, u)$ if for all $t \in T, f_0(t, \cdot) = f(t, \cdot)m$-a.e. on U. Then $X_0 = \{X_0(t) = \int_U f_0(t, u)\, W_m(du)\}$ is a version of X by (1.3). Define two conditions on the kernel $f(t, s)$:

(C1) f has a modification f_0 such that for every $u \in U$, $f_0(\cdot, u)$ is continuous.

(C2) $f^*(u) = \sup_{t \in T} |f(t, u)|$ is in $L^p(U, \mathscr{U}, m)$. (By $\sup |f(t, u)|$ we shall mean $\sup_{t \in T_0} |f(t, u)|$, where $T_0 \subset T$ is a countable separant for X that is dense in (T, τ).)

In [1], it is shown that if (C1) holds, then Itô and Nisio's [4] results on oscillation functions generalize to p-stable processes. This gives detailed information about what kind of paths such processes can have, but it does not give conditions on when those paths are continuous or bounded, nor indicate what happens when (C1) does not hold.

The next theorem resolves these questions when $0 < p < 1$. One surprising aspect of this is that there is no difference between the stationary and nonstationary case, unlike the Gaussian situation. When $1 \leqslant p < 2$, the situation is more like the Gaussian case, see Section 3.

THEOREM 1. *Let X be a real, symmetric, separable in probability p-stable process, $0 < p < 1$, on a compact metric or pseudo-metric space (T, τ).*

(i) *X has a version with a.s. continuous sample paths if and only if* (C1) *and* (C2) *hold for some (every) representation* (1.1).

(ii) *X has a version with a.s. unbounded sample paths if and only if* (C2) *fails to hold for some (every) representation* (1.1).

(iii) *X has a version with a.s. discontinuous, bounded sample paths if and only if* (C1) *fails to hold and* (C2) *does hold for some (every) representation* (1.1).

Proof. (i) Suppose (C1) and (C2) hold for some representation (1.1). Let f_0 be a version of f guaranteed by (C1) and set

$$N = \{u \in U: f(t_j, u) \neq f_0(t_j, u) \quad \text{for some } t_j \in T_0\}$$
$$= \bigcup_{j=1}^{\infty} \{u \in U: f(t_j, j) \neq f_0(t_j, u)\}.$$

This is a m-null set since f_0 is a modification of f. Thus for $u \notin N$,

$$f_0^*(u) = \sup_j |f_0(t_j, u)| = \sup_j |f(t_j, u)| = f^*(u).$$

Hence (C2) implies $f_0^* \in L^p(U, \mathscr{U}, m)$ also. Since $f_0(\cdot, u)$ is in $C(T)$ for all u, this says

$$\|f_0(\cdot, u)\|_{C(T)} = f_0^*(u) \in L^p(U, \mathscr{U}, m).$$

Applying the work of Marcus and Woyczynski [7] (see [10, Section 4]), $X_0 = \{\int_U f_0(t, u)\, W(du)\}$ has a.s. continuous paths.

Conversely, Theorem 5.1 of Rosinski [10] shows that X having a continuous version implies (C1) holds for every representation. Furthermore, Corollary 5.2 to Rosinski's theorem shows that a modification f_0 of f satisfies (C2), i.e., $f_0^* \in L^p(U, \mathscr{U}, m)$. The above argument shows $f^* = f_0^*$ m-a.e., so (C2) holds for f also.

(ii) By Theorem 6.2 of Samorodnitsky [11], (C2) is equivalent to X having a version with bounded paths when $0 < p < 1$. Again this result does not depend on the representation chosen.

(iii) Follows from (i) and (ii). ∎

The method of proving Theorem 1(i) applies to other Banach spaces besides $C(T)$. For example, let d be any pseudo-metric on T that is continuous with respect to τ, and define the possibly infinite function on $C(T)$:

$$\|f\|_{\mathrm{Lip}(d)} = \sup_{s, t \in T} \frac{|f(s) - f(t)|}{d(t, s)}.$$

Pick any $t_0 \in T$ and let $\mathrm{Lip}(d) = \{f \in C(T): \|f\|_{\mathrm{Lip}(d)} < \infty\}$. This is a Banach space with norm

$$\|f\| = |f(t_0)| + \|f\|_{\mathrm{Lip}(d)}.$$

Rephrasing (C1) and (C2) in terms of $\mathrm{Lip}(\tau)$ instead of $C(T)$ gives necessary and sufficient conditions for X to satisfy a Lipschitz condition.

COROLLARY 2. *Let X be as in Theorem* 1. *X has a version with paths in* $\mathrm{Lip}(\tau)$ *a.s. if and only if for some (every) representation* (1.1)

$$(\mathrm{Lip}(\tau)-1) \qquad f(t, u) \text{ has a version } f_0(t, u) \text{ with } f_0(\cdot, u) \in \mathrm{Lip}(\tau) \text{ for every } u,$$

and

$$(\mathrm{Lip}(\tau)-2) \qquad \|f_0(\cdot, u)\|_{\mathrm{Lip}(\tau)} \in L^p(U, \mathscr{U}, m).$$

3. CONTINUITY WHEN $1 \leqslant p < 2$

We now consider the cases when $1 \leqslant p \leqslant 2$. Let d be a metric or pseudo-metric on T and let q be the dual index of p, i.e., $p^{-1} + q^{-1} = 1$. The d-metric entropy is defined in the standard way: for $\varepsilon > 0$,

$$H_q(d; \varepsilon) = \begin{cases} (\log N(d; \varepsilon))^{1/q}, & 2 \leqslant q < \infty \\ \log^+ \log N(d; \varepsilon), & q = \infty, \end{cases}$$

where $N(d; \varepsilon) = N(T, d; \varepsilon) =$ minimum number of d-balls of radius ε with centers in T that cover T.

A particular pseudo-metric that is naturally associated with a stable process X is

$$\begin{aligned} d_X(t, s) &= (-\log[E \exp(i(X(t) - X(s))])^{1/p} \\ &= \|f(t, \cdot) - f(s, \cdot)\|_{L^p(U, \mathscr{U}, m)}. \end{aligned}$$

The last equality comes from (1.2) and shows that d_X and $H_q(d_X; \varepsilon)$ are independent of which representation (1.1) we are considering.

THEOREM 3. *Let $X = \{X(t),\ t \in T\}$ be a real, symmetric, separable in probability p-stable process, $1 \leqslant p < 2$, on a compact metric or pseudo-metric space (T, τ).*

(i) *If X has a version with a.s. continuous paths, then* (C1) *and* (C2) *hold for every representation* (1.1). *Furthermore, when $p > 1$,*

$$\lim_{\varepsilon \downarrow 0} \varepsilon H_q(d_X; \varepsilon) = 0.$$

(ii) *Assume* (C1) *and* (C2) *hold for some representation* (1.1) *of X and that $\int_0^\infty H_q(d_X; \varepsilon)\, d\varepsilon < \infty$. If f_0 is a modification of f satisfying* (C1) *and*

$$\int \|f_0(\cdot, u)\|^p_{\mathrm{Lip}(d_X)}\, m(du) < \infty, \tag{3.1}$$

then X has a version with continuous sample paths.

Before proving Theorem 3, we would like to state the following conjectures.

Conjecture 1. Condition (3.1) can be dropped in Theorem 3(ii); i.e., (C1), (C2), and $\int_0^\infty H_q(d_X; \varepsilon)\, d\varepsilon < \infty$ imply X has continuous paths.

Conjecture 2. Assume T is a locally compact abelian group and X is stationary. X has a.s. continuous paths if and only if (C1), (C2), and $\int_0^\infty H_q(d_X; \varepsilon)\, d\varepsilon < \infty$ for some (every) representation.

Both conjectures are true for harmonizable stable processes (random Fourier transforms) by [5], where (C1) and (C2) are automatic. Counterexamples showing $\int_0^\infty H_q(d_X; \varepsilon)\, d\varepsilon < \infty$ is not sufficient for continuity, e.g., Remark 1.7 [5], do not take (C1) and (C2) into account. If $X(t)$ is stationary sub-Gaussian, i.e., $X(t) = Z^{1/2} Y(t)$, where Z is a $(p/2)$-stable positive r.v. and $Y(t)$ is stationary Gaussian, then X is continuous when and only when Y is continuous, which occurs when and only when $\int_0^\infty H_2(d_X; \varepsilon)\, d\varepsilon < \infty$, *not* $\int_0^\infty H_q(d_X; \varepsilon)\, d\varepsilon < \infty$. Initially, this seems to doom the above conjectures. However, Hardin shows that one representation for sub-Gaussian processes is to use the paths of $Y(t)$ as the kernel in (1.1), i.e.,

$$X(t) = \int_\Omega Y(t, \omega)\, W_P(d\omega).$$

For this representation, (C1) requires that Y is a.s. continuous, which is equivalent to the correct $\int_0^\infty H_2(d_X; \varepsilon)\, d\varepsilon < \infty$. So the conjectures are plausible.

The proof of Theorem 3 depends on the next result, which is an extension of Theorem 1.6 of Marcus and Pisier [6]. Basically, that result gives a sufficient condition for path continuity of stochastic integrals of $C(T)$ valued integrands. Proposition 4 rephrases that result in terms of some kernel $f(t, u)$ in (1.1).

We define a few more terms. For $2 \leqslant q \leqslant \infty$, $\delta > 0$ and a pseudo-metric d on T, the metric entropy integral on $(0, \delta)$ is

$$J_q(d; \delta) = \int_0^\delta H_q(d; \varepsilon)\, d\varepsilon.$$

The d-diameter of T is $\hat{d} = \sup_{s,t \in T} d(s, t)$. Define for $v > 0$,

$$\phi_q(v) = \begin{cases} v(\log^+ \log(1/v))^{1/q}, & 2 \leqslant q < \infty \\ v(\log^+ \log^+ \log(1/v)), & q = \infty. \end{cases}$$

For real random variables Y in the weak $L_{p,\infty}$ spaces, we will use the function $\Lambda_p(Y) = \sup_{\lambda > 0} (\lambda^p P(|Y| > \lambda))^{1/p}$.

For the next result, (T, d) will be the pseudo-metric space of concern, not the original (T, τ) we have dealt with so far. In particular, $C(T)$ stands for functions that are continuous with respect to d; hence, (C1) should be interpreted in this sense.

PROPOSITION 4. *Let $X=\{X(t),\ t\in T\}$ be a real, symmetric, separable in probability p-stable process, $1\leqslant p<2$, on a compact metric or pseudo-metric space (T, d). Assume* (C1) *and* (C2) *hold for some representation* (1.1), *that $J_q(d;\delta)<\infty$ for some $\delta>0$ $(p^{-1}+q^{-1}=1)$ and that for a version f_0 of the kernel guaranteed by* (C1),

$$K(p,d)=\left(\int_U \|f_0(\cdot,u)\|^p_{\mathrm{Lip}(d)}\, m(du)\right)^{1/p}<\infty.$$

Then X has a version Y with a.s. continuous (with respect to d) sample paths satisfying

$$\Lambda_p\Big(\sup_{\substack{d(s,t)\leqslant\delta\\ s,t\in T}} |Y(S)-Y(t)|\Big)\leqslant c(p)\, K(p,d)[J_q(d;\delta)+\hat{d}\phi_q(\delta/4\hat{d})]$$

for some constant $c(p)$ depending only on p.

Proof. Let f_0 be a version of the kernel f that satisfies our hypothesis. We will define a normalized representation in terms of f_0. Pick any $h_1\in C(T)$ with $\|h_1\|_{C(T)}=1$ and define a new kernel

$$h(\cdot,u)=\begin{cases} h_1(\cdot), & \|f_0(\cdot,u)\|=0\\ \dfrac{f_0(\cdot,u)}{\|f_0(\cdot,u)\|_{C(T)}}, & \text{otherwise,}\end{cases}$$

and a new measure

$$\mu(du)=\|f_0(\cdot,u)\|^p_{C(T)}\, m(du).$$

Then $Y(t)=\int_U h(t,u)\, W_\mu(du)$ is a version of X because of (1.3). This representation in terms of h has the properties:

$$h(\cdot,u) \text{ is continuous for every } u; \tag{3.2}$$

$$\mu(U)=\int_U \|f_0(\cdot,u)\|^p_{C(T)}\, m(du)<\infty \qquad \text{by (C2);} \tag{3.3}$$

$$\begin{aligned} h^*(u)&=\|h(\cdot,u)\|_{C(T)}\\ &=1, \qquad \text{hence}\quad h^*\in L^p(U,\mathscr{U},\mu) \text{ by (3.3);}\end{aligned} \tag{3.4}$$

$$J_q(d; \delta) < \infty; \tag{3.5}$$

$$\int_U \|h(\cdot, u)\|^p_{\mathrm{Lip}(d)}\, \mu(du) < \infty; \tag{3.6}$$

since for each u,

$$\|h(\cdot, u)\|_{\mathrm{Lip}(d)} = \|f_0(\cdot, u)\|_{\mathrm{Lip}(d)}/\|f_0(\cdot, u)\|_{C(T)}$$

and $\mu(du) = \|f_0(\cdot, u)\|^p_{C(T)}\, m(du)$. In fact, the integral (3.6) is exactly $K(p, d)$. We will now use (3.2)–(3.5) to define a finite measure ν on $(C(T), \mathrm{Borel}(C(T)))$ with support on the unit sphere of $C(T)$. Since $\mathrm{Borel}(C(T))$ coincides with the cylindrical sigma-field on $C(T)$, it suffices to define ν on cylinder sets of the form $C = C(B_n; t_1, ..., t_n) = \{g \in C(T)\colon (g(t_1), ..., g(t_n)) \in B_n\}$, where $n \geqslant 1$, $t_1, ..., t_n \in T$ and $B_n \in \mathrm{Borel}(\mathbb{R}^n)$. For such a set C, define $\nu(C) = \mu\{u \in U\colon (h(t_1, u), ..., h(t_n, u)) \in B_n\}$; the latter set is $\mathscr{U}$-measurable since each $h(t_j, \cdot)$ is $\mathscr{U}$-measurable. Next symmetrize ν; let $\nu_{\mathrm{sym}} = \nu * \nu^*$ be the usual symmetrization. This is equivalent to looking at the measure induced on $C(T)$ by the kernel $h_{\mathrm{sym}}(t, (u, \varepsilon)) = \varepsilon h(t, u)$ on $T \times (U \times \{-1, 1\})$ and $U \times \{-1, 1\}$ has measure $\mu_{\mathrm{sym}} = \mu \times (\delta_{-1} + \delta_1)$. But the process $X_{\mathrm{sym}}(t) = \int h_{\mathrm{sym}}(t, \cdot)\, d\mu_{\mathrm{sym}}$ corresponds to $X(t) - X'(t)$, where X' is an independent copy of X. Since X is symmetric, X_{sym} is a version of X and we may replace ν by ν_{sym}.

We now have a finite, symmetric measure ν on the boundary of the unit ball of $C(T)$. Let M_ν be the p-stable noise generated by ν on $C(T)$ and define

$$Z(t) = \int_{C(T)} x(t)\, M_\nu(dx)$$

as in the discussion preceding Theorem 1.6 of [6]. This is a version of X also. Condition (3.5) is unchanged and condition (3.6) can be rephrased as

$$\int_{C(T)} \|x\|^p_{\mathrm{Lip}(d)}\, \nu(dx) < \infty. \tag{3.7}$$

Now apply Theorem 1.6 of [6] to conclude that Z, and hence X, has a version with continuous paths. (Note that [6] left out the condition (3.7) in the statement of their theorem.) ∎

Proof of Theorem 3. (i) As in Theorem 1(i), (C1), and (C2) hold for every representation. Theorem 2.6 of [5] shows $\lim_{\varepsilon \downarrow 0} \varepsilon H_q(d_X; \varepsilon) = 0$ when $p > 1$.

(ii) Let f_0 be a modification of the kernel f that satisfies (C1) and (3.1). We will show that $X_0(t) = \int_U f_0(t, u)\, W_m(du)$ has a continuous version. First we note that (C1) and (C2) imply $d_X(t, s) \to 0$ as

$\tau(t, s) \to 0$: (C1) implies $f_0(t, u) \to f_0(s, u)$ as $\tau(t, s) \to 0$ for each u, and $|f_0(t, u) - f_0(s, u)| \leqslant 2f_0^*(u)$, so (C2) and a dominated convergence argument show $d_X(t, s) = (\int |f(t, u) - f(s, u)|^p \, m(du))^{1/p} \to 0$ as $\tau(t, s) \to 0$. Thus it suffices to show X is a.s. continuous with respect to d_x. This follows from Proposition 4, which also gives a modulus of continuity. ▌

4. Path Continuity and Continuity at a Point

A Gaussian process with continuous covariance is path continuous if and only if it is continuous at each point. The stable analog follows.

THEOREM 5. *Let $X = \{X(t), t \in T\}$, be a p-stable metric or pseudo-metric space (T, τ), $0 < p < 2$. X has a version with a.s. continuous paths if and only if* (C1) *holds for some (every) representation and X is a.s. continuous at each point of T.*

Proof. Necessity is straightforward using Theorems 1 and 3. Sufficiency follows by assuming (C1) for some representation. Then the oscilllation function [1] of X is nonrandom. It is zero at a point t if and only if X is continuous at t. If X is continuous at each t, then the nonrandom oscillation function is identically zero and the process is path continuous. ▌

In this result and in the oscillation function results of [1], (C1) plays the role that the continuous covariance condition plays in the Gaussian case. Perhaps (C1) is the correct generalization of continuous covariance for stable processes, not simply that d_x is continuous. Recall from the proof of Theorem 3.2, (C1) and (C2) imply d_x is continuous with respect to τ.

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